<b>Step Response</b>

The system order is the same as the number of poles in the system. The poles determine the system response. Most systems can be approximated as a first or second order system. A first order system has the form of Equation (1). Where K is some constant, and P is pole location. A first order system has one of the two step responses shown in Figure (1) depending on the real part of the pole.

Equation : First order system.

Figure : First order responses

A second order system can be represented as Equation (2). Where <i>&omega;<sub>n</sub></i> is the natural frequency and <i>&zeta;</i> is the dampening coefficient.

Equation : Second order system.

This gives the poles of the system the following form.

A stable second order system will have 1 of 4 step responses shown in Figure (2).

Figure : Second order responses.

Having a critically damped system is ideal, because it gives the best performance. But this is hard to do, because the system model must be exact. It is usually assumed that the system is under damped because the systems response can be easily predicted. Figure (3) shows the second order response of the system.

Figure : Second order system response.

There are 5 characteristics of the second order system. The rise time (<i>t<sub>r</sub></i>) is the amount of time it takes for the output to reach the final value for the first time, see Equation (3).

Equation : Rise time equation.

The peak time (<i>t<sub>p</sub></i>) is amount of time it takes for the output to reach to peak value, see Equation (4).

Equation : Peak time equation.

The percent overshoot is how much the output overshoots the final value, see Equation (5).

Equation : Percent overshoot equation.

The maximum value is the largest value of the system output, see Equation (6).

Equation : Peak value equation.

The settling time is the amount of time it takes to reach 2% of its final value, see Equation (7).

Equation : Settling time equation.

Using these equations you can place the poles as needed to get the desired response.

<b>Steady State Error</b>

It’s almost always desirable to have zero steady state error in your control loop. In some cases it is desirable to be able to track a signal. More often it is required that the steady state or tracking error be within a certain amount. This can be predicted. The system error is shown in Equation (8) where <i>E</i> is the error, <i>R</i> is the reference signal and <i>Y</i> is the output.

Equation : System error.

The final value theorem relates limits in the time domain to limits in the frequency domain. We can use it to predict the final value in the time domain based on the initial value in the frequency domain. It is shown in Equation (9).

Equation : Final value theorem.

So using Equation (8) with (9) you get (10), which is used to find the steady state error for different inputs. Here <i>G(s)</i> is the plant, and <i>H(s)</i> is any dynamics present in the feedback loop.

Equation : Output error equation.

All you need is the Fourier transform of your input signal. Figure (4) shows common input signals and their Fourier transforms.

Figure : Different inputs in the frequency domain.